A Computational Signal Processing System for Correlated Digital Interferometry

Jorge Villamizar-Morales, Lola X. Bautista-Rozo, Domingo A. Rodriguez
Automated Information Processing Laboratory
Electrical and Computer Engineering Department
University of Puerto Rico at Mayaguez
Mayaguez, PR 00681-9042
e-mail:{Jorge.Villamizar,Lola.Bautista,Domingo}@ece.uprm.edu

***Abstract*— This work deals with the development of a computational signal algebra framework for the modeling and simulation of digital image interferometry processing applications, exploiting the rapid prototyping capabilities of MATLAB with the network computing capabilities of Java.** [1]

I. Introduction

The signal algebra is constructed by using the binary two-dimensional cyclic convolution as the product operation that turns a linear space of two-dimensional finite discrete images into a linear algebra. Matrix representations of two-dimensional cyclic convolution operations are represented as block circulant matrices with circulant blocks. This is accomplished when finite discrete image object arrays, serving as inputs in the matrix-vector computation of the two-dimensional cyclic convolution operation, are transformed into one-dimensional column vectors using, both, lexicographic and anti-lexicographic ordering. Special attention is given to the algebra of cyclic correlations which is related to the algebra of cyclic convolutions through the index reversal or reflection operator.

Correlated digital interferometry (DCI) for imaging radars deals with the use of signal correlation techniques to process the phase information of digital image representations of microwave imaging signals. Rapid prototyping of matrix computations are offered by MATLAB which makes it ideal for developing algorithms to model applications that usually involve large data sets. This work takes advantage of the Java Virtual Machine (JVM) offered by the MATLAB package to create and run programs that create and access Java objects.

II. Signal Algebra Framework

Given a finite set of finite signals, the linear signal algebra over the field $\mathbb{C}$ is defined as a Hilbert vector space denoted $(V, +, \cdot)$ with an additional operation called a vector multiplication or signal multiplication operation defined as follows:

$$* : V \times V \to V$$

$$(V_k, V_l) \mapsto *(V_k, V_l) = V_k * V_l = V_m \tag{1}$$

[1]Partially supported by NSF Grant No. CNS0424546

This signal multiplication operation has the following three properties: Associative, Distributive with respect to vector addition and Distributive Associative with respect to the scalars.

The standard basis set of this signal algebra is the set $\Delta_N \subset l\left(Z_{N_0 \times N_1}\right)$, of $N = N_0 \times N_1$ complex signals defined as

$$\Delta_N = \left\{ \begin{array}{r} \delta_{\{k_0,k_1\}} : \delta_{\{k_0,k_1\}}\left[n_0,n_1\right] = 1; \{k_0,k_1\} = [n_0,n_1] \\ \\ \delta_{\{k_0,k_1\}}\left[n_0,n_1\right] = 0; \{k_0,k_1\} \neq [n_0,n_1] \end{array} \right\} \tag{2}$$

A. The linear algebra of cyclic convolutions of order $N_0 \times N_1$ over $\mathbb{C}$

Given the vector space or signal space $l\left(Z_{N_0} \times Z_{N_1}\right)$, we turn this space into a linear algebra, which we call *Signal Algebra* by introducing the following vector multiplication or signal multiplication operation:

$$\bigotimes_{N_0 \times N_1} : l\left(Z_{N_0} \times Z_{N_1}\right) \times l\left(Z_{N_0} \times Z_{N_1}\right) \to \mathbb{C}$$

$$(x, h) \mapsto \otimes_N (x, h) = x \otimes_N h = y \tag{3}$$

1) Matrix representation of cyclic convolution operator $\bigotimes^h_{N_0 \times N_1}$ over $l\left(Z_{N_0 \times N_1}\right)$: The matrix representation of cyclic convolution operator using anti-lexicographic ordering with respect to the standard basis Δ_N is:

$$H_N = \left[\otimes^h_N \left\{\delta_{\{0,0\}}\right\} \ \otimes^h_N \left\{\delta_{\{1,0\}}\right\} \ldots \otimes^h_N \left\{\delta_{\{N_0-1,N_1-1\}}\right\}\right] \tag{4}$$

where $\delta_{\{i,j\}} \in \Delta_N$

$$\left(\bigotimes^h_N \left\{\delta_{\{k_0,k_1\}}\right\}\right)\left[n_0,n_1\right] = g_{\{k_0,k_1\}}\left[n_0,n_1\right] =$$

$$= \sum_{m_1}\sum_{m_0} \delta_{\{k_0,k_1\}}\left[m_0,m_1\right] h\left[\langle n_0 - m_0\rangle_{N_0}, \langle n_1 - m_1\rangle_{N_1}\right], \tag{5}$$

$m_0 \in Z_{N_0}, m_1 \in Z_{N_1}$

If $\{k_0, k_1\} = [n_0, n_1]$ then,

$$g_{\{k_0,k_1\}}[n_0, n_1] = h\left[\langle n_0 - k_0\rangle_{N_0}, \langle n_1 - k_1\rangle_{N_1}\right] \quad (6)$$

where $n_0, k_0 \in Z_{N_0}$ and $n_1, k_1 \in Z_{N_1}$

$$H_N = \left[\begin{array}{cccc} g_{\{0,0\}} & g_{\{1,0\}} & \dots & g_{\{N_0-1,N_1-1\}} \end{array}\right] =$$

$$\left[h\left[\langle n_0\rangle_{N_0}, \langle n_1\rangle_{N_1}\right] ...h\left[\langle n_0 - N_0 + 1\rangle_{N_0}, \langle n_1 - N_1 + 1\rangle_{N_1}\right]\right] \quad (7)$$

B. The linear algebra of cyclic correlations of order $N_0 \times N_1$ over $\mathbb{C}$

Given the vector space or signal space $l(Z_{N_0\times N_1})$, we turn this space into a linear algebra by introducing the following vector multiplication or signal multiplication operation:

$$\copyright_N : l(Z_{N_0\times N_1}) \times l(Z_{N_0\times N_1}) \to l(Z_{N_0\times N_1})$$
$$(x, h) \mapsto \copyright_{N_0\times N_1}(x, h) = x \copyright_{N_0\times N_1} h = y \quad (8)$$

1) Matrix representation of cyclic correlation operator $\copyright^g_{N_0\times N_1}$ over $N_0 \times N_1$: The matrix representation of cyclic correlation operator using anti-lexicographic ordering with respect to the standard basis Δ_N is:

$$C_N = \left[\copyright^g_N\left\{\delta_{\{0,0\}}\right\}\ \copyright^g_N\left\{\delta_{\{1,0\}}\right\}\dots\copyright^g_N\left\{\delta_{\{N_0-1,N_1-1\}}\right\}\right] \quad (9)$$

where $\delta_{\{i,j\}} \in \Delta_N$

$$\left(\copyright^g_N\left\{\delta_{\{k_0,k_1\}}\right\}\right)[n_0, n_1] = h_{\{k_0,k_1\}}[n_0, n_1] =$$

$$= \sum_{m_1}\sum_{m_0} \delta_{\{k_0,k_1\}}[m_0, m_1]\, g\left[\langle n_0 - m_0\rangle_{N_0}, \langle n_1 - m_1\rangle_{N_1}\right] \quad (10)$$

$m_0 \in Z_{N_0}, m_1 \in Z_{N_1}$

If $\{k_0, k_1\} = [n_0, n_1]$ then,

$$h_{\{k_0,k_1\}}[n_0, n_1] = g\left[\langle n_0 - k_0\rangle_{N_0}, \langle n_1 - k_1\rangle_{N_1}\right] \quad (11)$$

where $n_0, k_0 \in Z_{N_0}$ and $n_1, k_1 \in Z_{N_1}$

$$C_N = \left[\begin{array}{cccc} h_{\{0,0\}} & h_{\{1,0\}} & \dots & h_{\{N_0-1,N_1-1\}} \end{array}\right] =$$

$$\left[g\left[\langle n_0\rangle_{N_0}, \langle n_1\rangle_{N_1}\right] ...g\left[\langle n_0 - N_0 + 1\rangle_{N_0}, \langle n_1 - N_1 + 1\rangle_{N_1}\right]\right] \quad (12)$$

2) Hadamard Product Binary Operation over $l(Z_{N_0\times N_1})$: This operation is defined as follows:

$$\odot_{N_0\times N_1} : l(Z_{N_0\times N_1}) \to l(Z_{N_0\times N_1})$$

$$(v_k, v_l) \mapsto \odot_{N_0\times N_1}(v_k, v_l) = v \quad (13)$$

where,

$$v[n_0, n_1] = (\odot_{N_0\times N_1}(v_k, v_l))[n_0, n_1]$$

$$v[n_0, n_1] = (v_k[n_0, n_1], v_l[n_0, n_1]) \quad (14)$$

$n_0 \in Z_{N_0}$ and $n_1 \in Z_{N_1}$

3) Cyclic Reflection Operator or Cyclic Index Reversal Operator over $l(Z_N)$: The algebra of cyclic correlations is related with the algebra of the cyclic convolutions through the index reversal operator, which is defined as follows:

$$\Re_N : l(Z_N) \to l(Z_N)$$

$$x \mapsto \Re_N\{x\} \quad (15)$$

Where,

$$(\Re_N\{x\})[n_0, n_1] = x^{(-)}[n_0, n_1] = x\left[\langle -n_0\rangle_{N_0}, \langle -n_1\rangle_{N_1}\right] \quad (16)$$

The matrix representation of the cyclic reflection operator $\Re_N$ using anti-lexicographic ordering with respect to the standard basis is given by the following expression:

$$R_{N_0\times N_1} = \left[\Re_N\left\{\delta_{\{0,0\}}\right\}\ \Re_N\left\{\delta_{\{1,0\}}\right\}\dots\Re_N\left\{\delta_{\{N_0-1,N_1-1\}}\right\}\right] \quad (17)$$

C. Cyclic Shift Operator over $l(Z_{N_0\times N_1})$ with respect to the Standard Basis $\Delta_{N_0\times N_1}$

The cyclic shift operator $S_{N_0\times N_1}$ over the space $l(Z_{N_0\times N_1})$ with respect to the standard basis $\Delta_{N_0\times N_1}$ is defined as follows:

$$\mathscr{S}_{N_0\times N_1} : l(Z_{N_0\times N_1}) \to l(Z_{N_0\times N_1})$$

$$\delta_{k_0,k_1} \mapsto \mathscr{S}_{N_0\times N_1}\{\delta_{k_0,k_1}\} = \delta_{\langle k_0+1\rangle_{N_0}, \langle k_1+1\rangle_{N_1}} \quad (18)$$

The matrix representation of the shift operator $\mathscr{S}_N$ using anti-lexicographic ordering with respect to the standard basis is given by the following expression:

$$\mathscr{S}_{N_0\times N_1} = \left[\mathscr{S}_{N_0\times N_1}\left\{\delta_{\{0,0\}}\right\}\dots\mathscr{S}_{N_0\times N_1}\left\{\delta_{\{N_0-1,N_1-1\}}\right\}\right] \quad (19)$$

III. Correlated Digital Interferometry

Radar interferometry, as a correlation technique between two images, is used to detect Earth surface changes produced by phenomena such as landslides, earthquakes, and flash floods. Figure 2 describes the proposed model for the Correlated Digital Interferometry and which is explained below.

Let G be the finite set of any finite signals, say $G = \{G_i : G_i \in l(Z_{N_0} \times Z_{N_1}), i = 0, 1, 2, ..., K - 1\}$.

The operation of basic interferometry between any two element of G, say G_m, G_l is defined as the following Hadamard product:

$$G_m \odot_N G_l \quad (20)$$

where $N = N_0 \times N_1$

The operation of basic cyclic correlation between any two elements of G, say G_r, G_s is defined as the following Hadamard product:

$$G_r \copyright_N G^*_s = G_r \odot_N \Re_N(G^*_s) \quad (21)$$

where the symbol $*$ denotes conjugate operation.

By the Discrete Fourier Transform (DFT) properties we can establish an isomorphism between the linear algebra $(l(Z_N), +, \cdot, \otimes_N, \mathbb{C})$, called the algebra of cyclic convolutions and the linear algebra $(l(Z_N), +, \cdot, \odot_N, \mathbb{C})$, called the algebra of the Hadamard products. This isomorphism establishes the following duality between the object domain and the spectral domain:

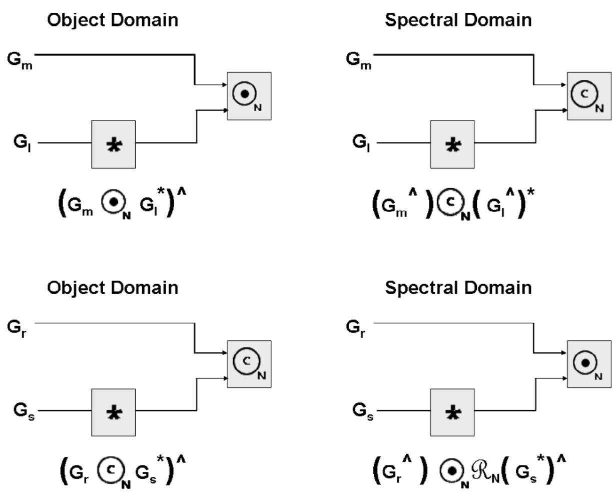

Fig. 1. Isomorphism between Linear Algebra of Cyclic Convolutions and the Algebra of Hadamard Product

Correlated Digital Interferometry (CDI) is defined as the cyclic correlation operation between any signals, that is:

$$\left((G_0 \odot_N G_1^*) \copyright_N \left((G_l \odot_N G_{l+1}^*)^*\right)\right)^{\frown} =$$

$$\left((G_0 \odot_N G_1^*) \otimes_N \Re_N \left((G_l \odot_N G_{l+1}^*)^*\right)\right)^{\frown} \qquad (22)$$

and

$$\left((G_0 \odot_N G_1^*) \otimes_N \Re_N \left((G_l \odot_N G_{l+1}^*)^*\right)\right)^{\frown} =$$

$$\left[1/N\widehat{G_0} \otimes_N \widehat{G_1^*}\right] \odot_N \Re_N \left[G_l^* \otimes_N G_{l+1}\right] \qquad (23)$$

Where the symbol $\frown$ denotes DFT.

IV. DCI Computational Environment

The conceptual model described previously has been implemented in a computational environment supported by the interfacing of $MATLAB^{®}$ and Java, taking advantage in the fact that the Java Virtual Machine of $MATLAB^{®}$ can interpret, create and access Java objects from the command prompt of $MATLAB^{®}$.

In the Java side there is a disbributed system tool-environment whose initial target is synthetic aperture radar (SAR) imaging applications. This tool-environment allows a given user to effect important image processing functions such as to visualize, manipulate, improve, filter, detect edges, and reduce the noise on SAR images. Also, this system has the unique option of allowing end-users to add their own customized algorithms as encapsulated operators to act on elements resident on local or remote SAR images-servers on a computer network. Now, special attention is being given to modify the environment in order to incorporate modules for the processing of DCI applications in distributed environments.

One of the important advantages of using Java is its network friendliness. The Java core contains many features that help us develop this network-based application because it takes into account issues as framentation, replication, naming, concurrency, failure, configuration and communication. In this work is used the client-server paradigm. With the client-server approach, multiple clients from a local or a remote machine can access the same application at any time.

In the $MATLAB^{®}$ side has been implemented the algorithms of Cyclic Convolution Operator, Cyclic Correlation Operator, Hadamard Product, Index Reversal Operator and Shift Operator as was proposed in the model. Even $MATLAB^{®}$ offers rapid prototyping of matrix computations, however its performance code does not support large data sets as SAR images. In this way, we are developing for future applications an interface for calling $MATLAB^{®}$ from Java in order to use a repository of algorithms that we have formulated and that require to be executed in a distributed system.

V. Conclusion

This work has devoloped a computational signal processing system for the modeling and simulation of Correlated Digital Interferometry (CDI) operations. Interferometry operations are prevalent in many scientific and engineering applications and the modality of corralated interferometry establishes a duality between the physical environment or object domain and the spectral domain through a Fourier homomorphism. Signal Algebra was instrumental in the formulation of the CDI theory. A Java-based environment was developed to serve as an interfaced between the CDI system and the application user. Hardware implementations of basic CDI system is contemplated for environmental survaillance applications.

Acknowledgment

We thank our students colleagues Carlos Huallparimachi and Ivan Rivera for their support in this work.

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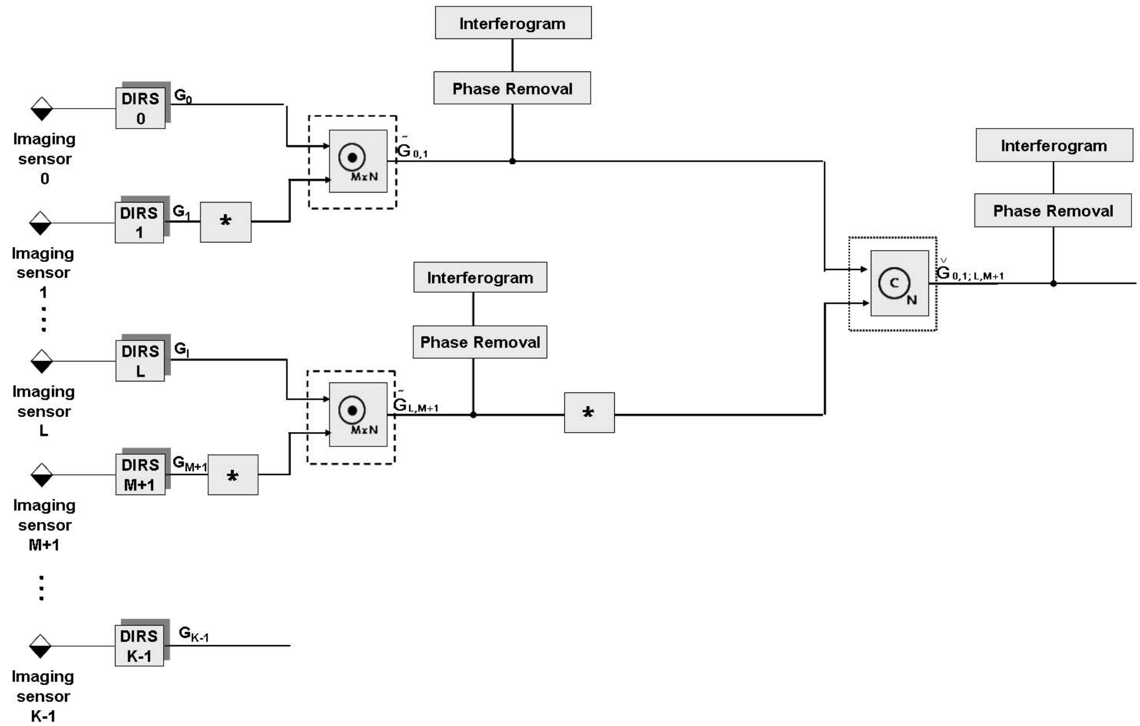

Fig. 2. Conceptual Model of the Correlated Digital Interferometry Process. Considering a set of sensors distributed spatially, the raw data acquired by means of them is processed in an equal number of Digital Image Representation Systems (DIRS), obtaining a set of finite discrete images denoted by $G_i, i = 0, 1, ..., K-1$. The process highlighted by the first two dashed boxes at the left side of the graph above, indicates the operation described in Equation (20) which corresponds to the Hadamard Product operator. In the same manner, the dashed box located at the right side of the graph above, indicates the operation described in Equation (21) which corresponds to the Cyclic Correlation operator.

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